Tensor-force effects on single-particle levels and proton bubble structure around the Z or $N = 20$ magic number

H. Nakada* and K. Sugiura
Department of Physics,
Graduate School of Science, Chiba University
Yayoi-cho 1-33, Inage, Chiba 263-8522, Japan

J. Margueron
Institut de Physique Nucléaire IN2P3-CNRS and Université Paris-Sud, F-91406 Orsay Cedex, France
(Dated: November 27, 2012)

Applying the semi-realistic NN interactions that include realistic tensor force to the Hartree-Fock calculations, we investigate tensor-force effects on the single-particle levels in the Ca isotopes. It is clarified that the tensor force is important in reproducing the experimental difference between $\varepsilon(p1s_{1/2})$ and $\varepsilon(p0d_{3/2})$ (denoted by $\Delta\varepsilon_{13}$) both at ^{40}Ca and ^{48}Ca. The tensor force plays a role in the N-dependence of $\Delta\varepsilon_{13}$ also in neutron-rich Ca nuclei. We further investigate possibility of proton bubble structure in Ar, which is suggested by the $p1s_{1/2}$-$p0d_{3/2}$ inversion in ^{48}Ca and more neutron-rich Ca nuclei, by the spherical Hartree-Fock-Bogolyubov calculations. Even with the inversion at ^{48}Ca the pair correlation prohibits prominent bubble distribution in ^{46}Ar. Bubble in Ar is unlikely also near neutron drip line because either of unboundness or of deformation. However, ^{34}Si remains a candidate for proton bubble structure, owing to large shell gap between $p1s_{1/2}$ and $p0d_{5/2}$.

PACS numbers: 21.10.Pc, 21.10.Ft, 21.60.Jz, 27.40.+z, 27.30.+t

Introduction. Owing to the progress in experiments on unstable nuclei, it has been recognized [1] that the nuclear shell structure depends on Z or N as often called *shell evolution*. Moreover, it is now known [2–4] that the tensor force, which should be contained in the nucleon-nucleon (NN) interaction, plays a crucial role in the shell evolution. While the tensor force had been ignored in the conventional mean-field (MF) or the energy-density-functional (EDF) approaches, there have been several attempts to incorporate the tensor force into the those approaches; *e.g.* the calculations with the Skyrme [5–8] or the Gogny [9] interactions. However, without well-established strengths and/or radial-dependence of the tensor force, it is not straightforward to pin down tensor-force effects quantitatively from those interactions. One of the authors (H.N.) has developed the so-called semi-realistic NN interactions [10], which is applicable to the MF calculations. The recent parameter-sets, M3Y-Pn with $n \geq 5$ [3, 4, 11], include the tensor force originating from the G-matrix at the nuclear surface. Because of the realistic nature of the tensor force in them, these semi-realistic interactions are suitable to investigate the tensor-force effects on the shell evolution.

Although the MF or EDF approaches give single-particle (s.p.) levels in a self-consistent manner, each of the s.p. levels does not correspond to a single observed state even beside the doubly magic nuclei, being fragmented over a certain energy range. The s.p. energies in the MF approaches should better correspond to the averaged energy of the states having the specific quantum numbers weighted by the spectroscopic factors. There are not many cases in which the spectroscopic factors have exhaustively been measured with good accuracy. The proton $1s_{1/2}$ and $0d_{3/2}$ hole states in ^{40}Ca and ^{48}Ca provide us with indispensable examples, for which sum of the measured spectroscopic factor exceeds 90% [12, 13]. These experimental data have disclosed a notable consequence that the $p1s_{1/2}$ and $p0d_{3/2}$ levels invert as N increases from ^{40}Ca to ^{48}Ca. The $p1s_{1/2}$-$p0d_{3/2}$ difference and their inversion seem to supply a good test of the MF effective interactions (or EDFs). Since it has been suggested that the tensor force plays a important role in this inversion [14, 15], it is of interest to apply the semi-realistic interaction. Moreover, the $p1s_{1/2}$-$p0d_{3/2}$ inversion at ^{48}Ca suggests a possibility of proton "bubble" structure, depletion of the proton density at the center of the nucleus, in the two-more-proton deficient nucleus ^{46}Ar [16]. It is noted that similar inversion has been predicted in several Ca nuclei near the neutron drip line, which suggests proton bubble in Ar. Possibility of proton bubble structure has been pointed out also for ^{34}Si [17]. Although it has been difficult to measure charge densities at the center of unstable nuclei, the new technology such as SCRIT [18] could open a way to observe such proton bubble structure.

In this paper we apply the self-consistent Hartree-Fock (HF) and Hartree-Fock-Bogolyubov (HFB) calculations with the semi-realistic interactions to the Si to Ca nuclei, and investigate tensor-force effects on the shell evolution and the bubble structure.

Effective Hamiltonian. We apply the spherical HF and HFB calculations by using the Gaussian expansion method [19, 20]. The details of the basis functions are given in Ref. [21]. The effective Hamiltonian

has the form $H = H_N + V_C - H_{\rm c.m.}$, where $H_N(= \sum_i \mathbf{p}_i^2/2M + \sum_{i<j} v_{ij})$, V_C and $H_{\rm c.m.}$ denote the effective nuclear Hamiltonian, the Coulomb interaction and the center-of-mass Hamiltonian, respectively. It is noted that the exchange term of V_C is treated exactly and that both the one- and the two-body terms of $H_{\rm c.m.}$ are subtracted before iteration.

The M3Y-Pn semi-realistic interactions have been obtained by modifying the so-called M3Y-Paris interaction [22], which was derived by fitting the Yukawa functions to the G-matrix at the nuclear surface. Density-dependent contact terms have been added to the M3Y-Paris interaction so as to realize the saturation, and the LS channels have been enhanced in order to reproduce the ℓs splitting at the MF level. We employ the M3Y-P5$'$ [23] and M3Y-P7 [11] parameter-sets in the following. In both sets the tensor force of the M3Y-Paris interaction is maintained without any change, as in M3Y-P5 [3] and P6 [11]. For comparison we also use the D1M [24] parameter-set of the Gogny interaction, which has no tensor channels. Whereas we have implemented calculations with M3Y-P6 and D1S [25], results of M3Y-P6 (D1S) are similar to those of M3Y-P7 (D1M).

N-dependence of $p1s_{1/2}$-$p0d_{3/2}$ levels in $Z = 20$ nuclei. We here express the s.p. energy difference under interest as $\Delta\varepsilon_{13} = \varepsilon(p1s_{1/2}) - \varepsilon(p0d_{3/2})$. Figure 1 shows N-dependence of $\Delta\varepsilon_{13}$ in the Ca isotopes obtained by the spherical HF calculations. The experimental values of $\Delta\varepsilon_{13}$ in ^{40}Ca and ^{48}Ca are also displayed for comparison, which are obtained after average weighted by the spectroscopic factors [12, 13]. Although significant N-dependence is found in the experiments on $\Delta\varepsilon_{13}$ of the Ca isotopes, not many MF interactions (or EDFs) can reproduce this N-dependence quantitatively [14]. In practice, despite agreement of $\Delta\varepsilon_{13}$ with the data at ^{48}Ca, there is significant discrepancy at ^{40}Ca in the D1M result, as viewed in Fig. 1. With D1S, $\Delta\varepsilon_{13}$ is slightly shifted downward and the inversion at ^{48}Ca does not occur. The slope of $\Delta\varepsilon_{13}$ from ^{40}Ca to ^{48}Ca in the D1M result is typical to the MF calculations with no tensor force. On the contrary, as depicted in Fig. 1, the semi-realistic M3Y-P5$'$ and P7 interactions successfully reproduce the N-dependence of $\Delta\varepsilon_{13}$. In Fig. 1 we also show $\Delta\varepsilon_{13}$ in which contribution of the tensor force is removed from the M3Y-P7 result. Then $\Delta\varepsilon_{13}$ varies in parallel to those of D1M, unable to reproduce the observed slope. This confirms the crucial role of the tensor force in $\Delta\varepsilon_{13}$ as $n0f_{7/2}$ is occupied.

The variation of $\Delta\varepsilon_{13}$ from $N = 20$ to 28 is a result of occupation of the $n0f_{7/2}$ orbit. As $n0f_{7/2}$ is occupied, the tensor force tends to lower $p0d_{3/2}$ but not $p1s_{1/2}$, therefore increasing $\Delta\varepsilon_{13}$. In D1M the parameters in the central and LS channels have globally been adjusted to experimental data, and the inversion of the s.p. levels at ^{48}Ca is reproduced. However, because lacking the tensor force, this inversion takes place at the expense of the discrepancy in $\Delta\varepsilon_{13}$ at ^{40}Ca. It should be commented here that there might be influence of ground-state correlations on $\Delta\varepsilon_{13}$ which are not well taken into account in the averaged s.p. energies, and that its assessment is desired for complete understanding.

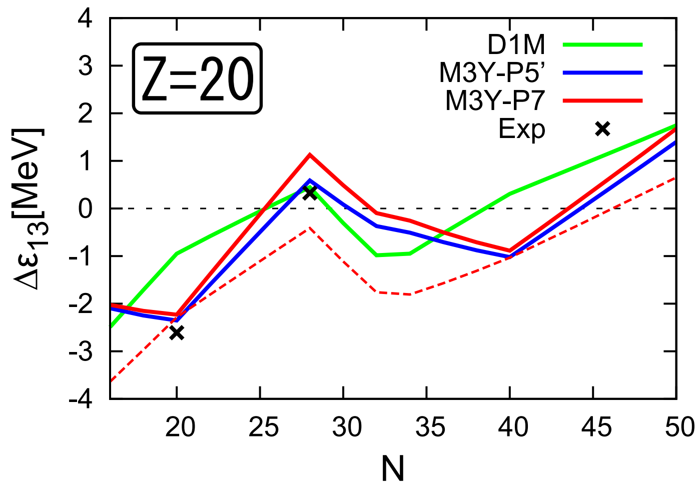

FIG. 1. (Color online) $\Delta\varepsilon_{13}$ of the Ca isotopes. Green, blue and red lines represent the results with the D1M, M3Y-P5$'$ and P7 interactions, respectively. Thin red dashed line is obtained from M3Y-P7 but by removing contribution of the tensor force.

The tensor force affects $\Delta\varepsilon_{13}$ from $N = 34$ to 40, *i.e.* as $n0f_{5/2}$ is occupied. Experiments on the $p1s_{1/2}$ and the $p0d_{3/2}$ hole states around ^{60}Ca may provide further evidence of the tensor-force effects, if carried out in the future. Although it depends on the effective interactions even in MF approaches where the neutron drip line of Ca is, $p1s_{1/2}$-$p0d_{3/2}$ inversion could occur again toward ^{70}Ca as $n0g_{9/2}$ is occupied. However, whereas the inversion takes place already at ^{60}Ca with D1M, the inversion is delayed in the M3Y-P5$'$ and P7 results, in which the parameters have been determined in the presence of the tensor force.

Investigation of proton bubbles. The $p1s_{1/2}$-$p0d_{3/2}$ inversion in ^{48}Ca indicates that dominant configuration of the two-proton-deficient nucleus ^{46}Ar is $(p1s_{1/2})^{-2}$. Since only the s-states give sizable density at the center of nuclei, ^{46}Ar is a candidate of a nucleus having proton bubble structure [16]. While it is not easy to measure matter or neutron densities in a model-independent manner particularly for unstable nuclei, charge densities may unambiguously be extracted from the electron scattering experiments. Since the M3Y-Pn semi-realistic interactions successfully describe the N-dependence of $\Delta\varepsilon_{13}$, indicating that mechanism giving rise to the $p1s_{1/2}$-$p0d_{3/2}$ inversion is correctly contained, it will be of certain interest to investigate possibility of proton bubble structure by applying these interactions.

We present proton density distributions at ^{48}Ca and ^{46}Ar obtained from the MF calculations with D1M and

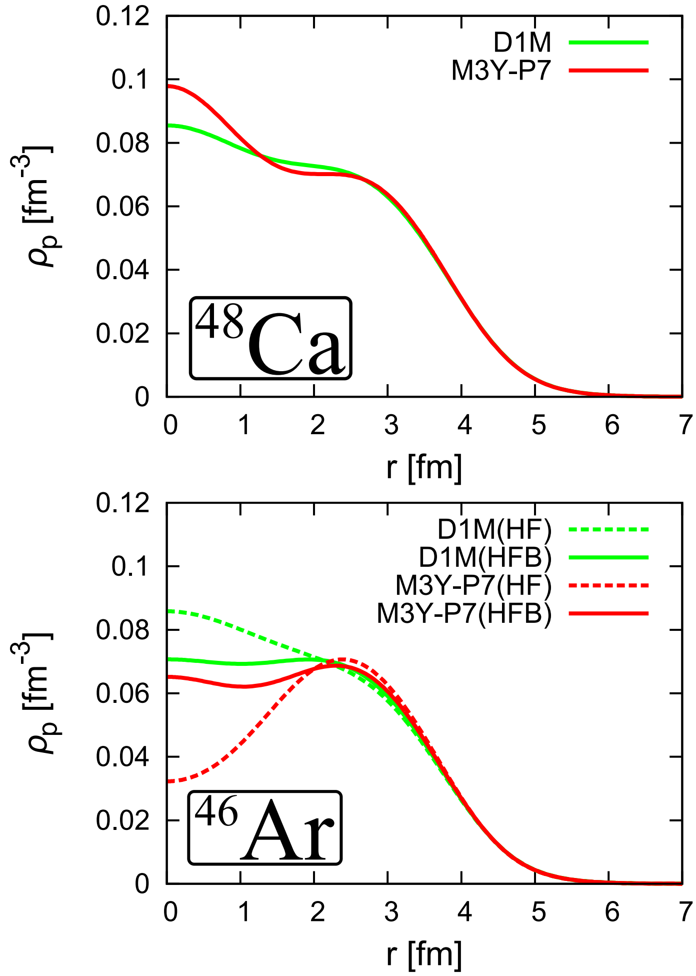

FIG. 2. (Color online) Proton density distributions in ^{48}Ca and ^{46}Ar obtained from HF and HFB calculations.

M3Y-P7 in Fig. 2. The pair correlation is quenched in the ground-state of ^{48}Ca and the HF and HFB results are identical. Within the spherical HF regime with M3Y-P7, we have depletion of the proton distribution around the origin at ^{46}Ar, since $p1s_{1/2}$ becomes unoccupied if compared with ^{48}Ca. The same holds for M3Y-P5$'$, though not displayed. Such depletion is not found in the HF result with D1M, in which the ground state has the $(p0d_{3/2})^{-2}$ configuration. Despite the $p1s_{1/2}$-$p0d_{3/2}$ inversion at ^{48}Ca, the total energy of the $(p0d_{3/2})^{-2}$ state becomes lower than that of the $(p1s_{1/2})^{-2}$ state at ^{46}Ar in the D1M result. On the other hand, once the pair correlation is taken into account, small energy difference between $p1s_{1/2}$ and $p0d_{3/2}$ significantly mixes up the $(p1s_{1/2})^{-2}$ and the $(p0d_{3/2})^{-2}$ configurations for both interactions. Thus the depletion observed in the HF densities with M3Y-P7 is smoothed out when the pairing is switched on.

While one might consider the formation of proton bubbles around ^{60}Ca because of the $p1s_{1/2}$-$p0d_{3/2}$ inversion given by D1M, the tensor force prohibits the inversion and accordingly the proton bubble structure, as indicated by Fig. 1. As previously discussed this is mostly due to the occupation of $n0f_{5/2}$. In contrast, the inversion is predicted by all the interactions under investigation around ^{70}Ca. The ^{70}Ca nucleus is bound in the HFB calculations with D1M, M3Y-P5$'$ and P7 [11, 23]. At a glance, the $p1s_{1/2}$-$p0d_{3/2}$ inversion exhibited in Fig. 1 seems to suggest proton bubble structure in $^{64-68}$Ar even with M3Y-P7. However, the $^{64-68}$Ar nuclei lie beyond the neutron-drip line while $^{66-70}$Ca are bound, within the spherical HFB calculation. Although these Ar nuclei might be deformed and thereby their energy could be lowered enough to be bound, the deformation significantly mixes the $(p1s_{1/2})^{-2}$ state with others, which easily destroys bubble structure. We therefore conclude that proton bubble structure is unlikely to be observed in any of the Ar nuclei.

While the $N = 20$ magic number disappears in the proton-deficient region of $Z \leq 12$, it remains magic in $Z \geq 14$, keeping the nuclear shape spherical. Possibility of proton bubble structure has been pointed out also for ^{34}Si [17]. The energy difference $\Delta\varepsilon_{15} = \varepsilon(p1s_{1/2}) - \varepsilon(p0d_{5/2})$ exceeds 5 MeV for ^{36}S in the spherical HF calculations, irrespective of the effective interactions under consideration. We note that $-\Delta\varepsilon_{13}$ is less than 2 MeV, giving rise to sizable pair excitation at ^{36}S. The occupation probability of $p1s_{1/2}$ is $0.6-0.7$ in the spherical HFB results. However, owing to the large subshell gap $\Delta\varepsilon_{15}$, the ground state of ^{34}Si is expected to be $(p1s_{1/2})^{-2}$ with good approximation. Proton density distribution of ^{34}Si is depicted in Fig. 3 in comparison with that of ^{36}S. Since $\Delta\varepsilon_{15}$ is sufficiently large, prominent proton bubble structure is predicted in ^{34}Si in the MF calculations for all the effective interactions considered in this paper. The large $\Delta\varepsilon_{15}$ prevents the pair correlation from mixing the $(p1s_{1/2})^{-2}$ state with the other states (*e.g.* $(p0d_{5/2})^{-2}$) in the ground state of ^{34}Si, giving identical density distribution between HF and HFB. Although influence beyond the MF regime is not obvious [26], ^{34}Si remains to be a candidate for the proton bubble. Future experiments on the charge density of this nucleus are awaited.

Summary. We have investigated tensor-force effects on the shell evolution and the bubble structure around the Z or $N = 20$ magic number, by applying the self-consistent HF and HFB calculations with the semi-realistic interactions. As well as having confirmed significance of the tensor force in the N-dependence of the $p1s_{1/2}$-$p0d_{3/2}$ difference in the Ca isotopes, we have shown that realistic tensor force gives adequate N-dependence if combined with appropriate central and LS forces, as in the M3Y-Pn semi-realistic interactions. Although the $p1s_{1/2}$-$p0d_{3/2}$ inversion is predicted for the Ca nuclei near the neutron drip line, the inversion is delayed with the interactions including realistic tensor force, suggesting the inversion only in $^{66-70}$Ca.

On the basis that semi-realistic interactions reproduce

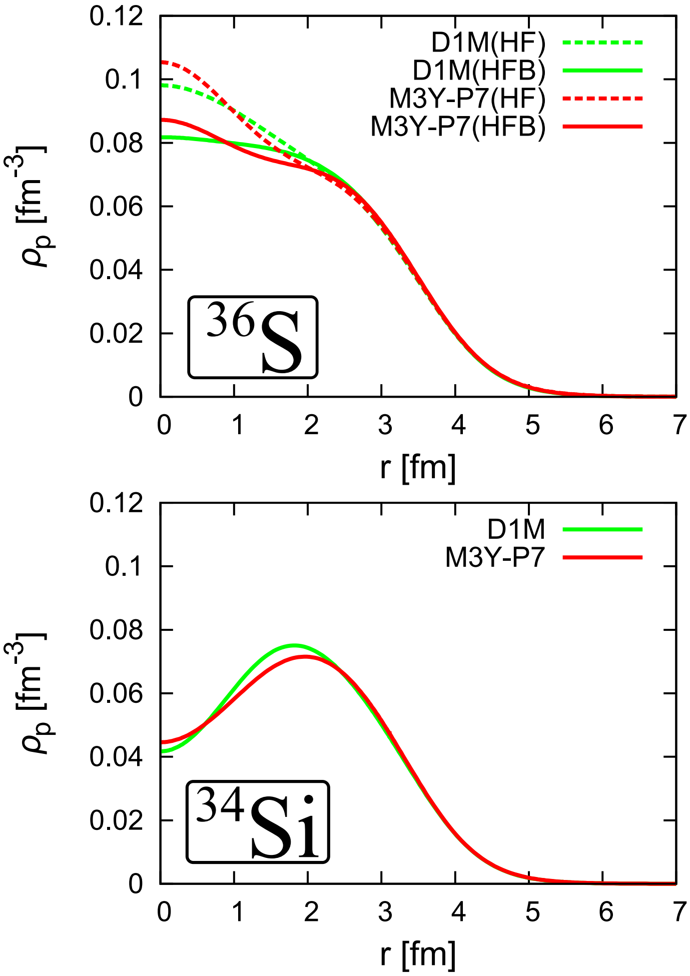

FIG. 3. (Color online) Proton density distributions in ^{36}S and ^{34}Si obtained from HF and HFB calculations.

the observed $p1s_{1/2}$-$p0d_{3/2}$ inversion reasonably well, we apply them to perform predictions for the proton-bubble-structure problem. In ^{46}Ar, although we view depletion of the proton density within HF, it is predicted that pair correlation prohibits the bubble structure from being realized. Although one may anticipate proton bubble structure in the highly neutron-excess nuclei $^{64-68}$Ar because of the inversion in $^{66-70}$Ca, these nuclei might be unbound or deformed, and therefore proton bubble structure is unlikely. On the contrary, it may remain hopeful that the proton bubble structure will be observed in future experiments in ^{34}Si.

This work is financially supported as Grant-in-Aid for Scientific Research (C), No. 22540266, by JSPS. Numerical calculations are performed on HITAC SR16000s at Institute of Media and Information Technology in Chiba University, Yukawa Institute for Theoretical Physics in Kyoto University, Research Institute for Information Technology in Kyushu University, Information Technology Center in University of Tokyo, and Information Initiative Center, Hokkaido University.

* E-mail: nakada@faculty.chiba-u.jp

[1] O. Sorlin and M.-G. Porquet, Prog. Part. Nucl. Phys. **61**, 602 (2008).
[2] T. Otsuka, T. Suzuki, R. Fujimoto, H. Grawe and Y. Akaishi, Phys. Rev. Lett. **95**, 232502 (2005).
[3] H. Nakada, Phys. Rev. C **78**, 054301 (2008); *ibid.* **82**, 029902(E).
[4] H. Nakada, Phys. Rev. C **81**, 027301 (2010); *ibid.* **82**, 029903(E).
[5] B.A. Brown, T. Duguet, T. Otsuka, D. Abe and T. Suzuki, Phys. Rev. C **74**, 061303(R) (2006).
[6] G. Colò, H. Sagawa, S. Fracasso and P.F. Bortignon, Phys. Lett. B **646**, 227 (2007).
[7] D.M. Brink and Fl. Stancu, Phys. Rev. C **75**, 064311 (2007).
[8] T. Lesinski, M. Bender, K. Bennaceur, T. Duguet and J. Meyer, Phys. Rev. C **76**, 014312 (2007).
[9] T. Otsuka, T. Matsuo and D. Abe, Phys. Rev. Lett. **97**, 162501 (2006).
[10] H. Nakada, Phys. Rev. C **68**, 014316 (2003).
[11] H. Nakada, submitted to Phys. Rev. C (arXiv:1206.0794).
[12] P. Doll, G.J. Wagner, K.T. Knöpfle and G. Mairle, Nucl. Phys. A **263**, 210 (1976).
[13] C.A. Ogilvie *et al.*, Nucl. Phys. A **465**, 445 (1987).
[14] M. Grasso, Z.Y. Ma, E. Khan, J. Margueron and N. Van Giai, Phys. Rev. C **76**, 044319 (2007).
[15] Y.Z. Wang, J.Z.Gu, X.Z. Zhang and J.M. Dong, Phys. Rev. C **84**, 044333 (2011).
[16] E. Khan, M. Grasso, J. Margueron and N. Van Giai, Nucl. Phys. A **800**, 37 (2008).
[17] M. Grasso *et al.*, Phys. Rev. C **79**, 034318 (2009).
[18] T. Suda *et al.*, Phys. Rev. Lett. **102**, 102501 (2009).
[19] H. Nakada and M. Sato, Nucl. Phys. A **699**, 511 (2002); *ibid.* **714**, 696 (2003).
[20] H. Nakada, Nucl. Phys. A **764**, 117 (2006); *ibid.* **801**, 169 (2008).
[21] H. Nakada, Nucl. Phys. A **808**, 47 (2008).
[22] N. Anantaraman, H. Toki and G.F. Bertsch, Nucl. Phys. A **398**, 269 (1983).
[23] H. Nakada, Phys. Rev. C **81**, 051302(R) (2010).
[24] S. Goriely, S. Hilaire, M. Girod and S. Pèru, Phys. Rev. Lett. **102**, 242501 (2009).
[25] J.F. Berger, M. Girod and D. Gogny, Comp. Phys. Comm. **63**, 365 (1991).
[26] J.-M. Yao, S. Baroni, M. Bender and P.-H. Heenen, Phys. Rev. C **86**, 014310 (2012).